

Magnetic field generation from cosmological perturbations

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Abstract. We discuss generation of magnetic field from cosmological perturbations. We consider the evolution of three component plasma (electron, proton and photon) evaluating the collision term between electrons and photons up to the second order. The collision term is shown to induce electric current, which then generate magnetic field. There are three contributions, two of which can be evaluated from the first-order quantities, while the other one is fluid vorticity which is purely second order. We compute numerically the magnitudes of the former contributions and shows that the amplitude of the produced magnetic field is about $\sim 10^{-19}$ G at 10kpc comoving scale at present. Compared to astrophysical and inflationary mechanisms for seed-field generation, our study suffers from much less ambiguities concerning unknown physics and/or processes.

Key words: magnetic field — cosmology — perturbation

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1. Introduction

There are convincing evidences that imply existence of substantial magnetic fields in various astronomical objects. Not only galaxies but systems with even larger scales, such as cluster of galaxies and extra-cluster fields, have their own magnetic fields (for a review on cosmological magnetic fields, see e.g., [1]). Conventionally the magnetic fields in galaxies, and possibly in clusters of galaxies, are considered to have been amplified and maintained by dynamo mechanism. However, the dynamo mechanism needs the seed magnetic field and does not explain the origin of the magnetic fields.

There have been many attempts to generate the seed fields. One of the approaches to this problem is to generate magnetic fields astrophysically, often involving the Biermann mechanism [2]. This mechanism has been applied to various systems: large-scale structure formation [3], ionizing front [4], protogalaxies [5] and supernova remnant of the first stars [6]. These studies showed the possibilities of magnetic-

field generation with amplitudes of order $10^{-16} \sim 10^{-21}$ G, which would be enough for the required field $\sim 10^{-20}$ G.

On the other hand, cosmological origins, which are often concerned with inflation, can produce magnetic field with coherence lengths of much larger scales, which is typically the horizon scale or possibly super-horizon scale [7, 8, 9, 10, 11]. Also they can often produce fields with a wide range of length scales, while astrophysical mechanisms can produce fields only with their characteristic scales. For a constraints on magnetic field with cosmological scale, see [12].

Here, we consider magnetic-field generation from cosmological perturbations after inflation. Around and after the decoupling, coupling among charged particles and photons become so weak that electric current can be induced by the difference in motions of protons and electrons. This electric current leads to generation of magnetic fields. It is well known that vorticity of plasma produces such electric current [13, 14, 15, 16, 17, 18]. However, because vorticity is not produced at the first order in cosmological perturbations, we must study the second order. We will consider equations of motion for protons, electrons and photons separately up to the second order, although equation of motion for photons does not appear explicitly. To study electric current appro-

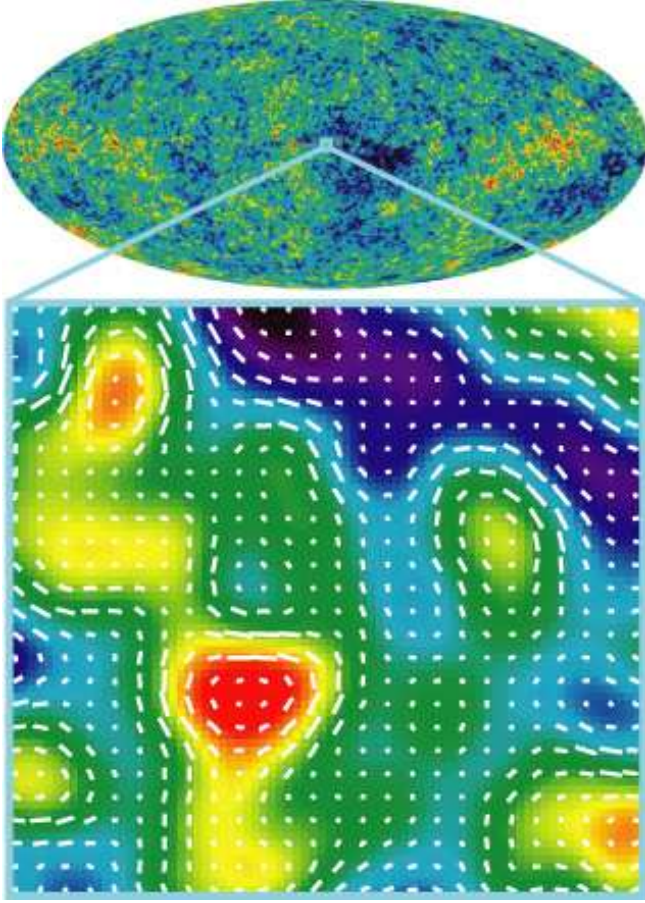


Fig. 1. All sky map of cosmological microwave background anisotropy obtained by the Wilkinson Microwave Anisotropy Probe (WMAP) satellite (upper) and schematic picture of cosmological magnetic fields generated from density fluctuations discussed here. Red (blue) regions are hot (cold) spots with a range of temperature $\sim 2.725 \text{ K} \pm 200 \mu \text{ K}$. The magnetic field vectors are shown together with the map. Strong magnetic fields are generated by currents where the gradient of density perturbation in photons is large.

privately, we must treat the three components separately [17]. Furthermore, we will evaluate the collision term between electrons and photons, which is dominant and essential for the magnetic-field generation, up to the second order. From the equations of motion for protons and electrons, we obtain a generalized Ohm's law, which, combined with the Maxwell equations, leads to an evolution equation for magnetic field. Our study is based on the cosmological perturbation theory, which is highly successful in the anisotropies of the cosmic microwave background, and suffers from much less ambiguities concerning unknown physics and/or processes compared to astrophysical and inflationary mechanisms for seed-field generation. For details of our study, see [19, 20].

2. Formulation

Euler equations for proton fluid and electron fluid are given by

$$(\delta_\mu^i + u^i u_\mu) (T_p^{\mu\nu} + T_{\text{EM}p;\nu}^{\mu\nu}) = C_{pe}^{(C)i} + C_{p\gamma}^{(T)i}, \quad (1)$$

$$(\delta_\mu^i + u^i u_\mu) (T_e^{\mu\nu} + T_{\text{EM}e;\nu}^{\mu\nu}) = C_{ep}^{(C)i} + C_{e\gamma}^{(T)i}, \quad (2)$$

where $T_p^{\mu\nu}$ and $T_{\text{EM}p(e)}^{\mu\nu}$ are the energy-momentum tensor of proton (electron) fluid and electromagnetic field coupling to protons (electrons) current, respectively. Here $\mu, \nu = 0, 1, 2, 3$ and $i = 1, 2, 3$. The projection of the divergence of the energy-momentum tensors are computed as

$$(\delta_\mu^i + u^i u_\mu) T^{\mu\nu}_{;\nu} = (\rho + p) u^\mu u^i_{;\mu} + (\delta_\mu^i + u^i u_\mu) P^{;\mu}, \quad (3)$$

$$(\delta_\mu^i + u^i u_\mu) T_{\text{EM};\nu}^{\mu\nu} = -j^\nu F^\mu_\nu, \quad (4)$$

where ρ , P and j^μ are the energy density, pressure and electric current, respectively. The r.h.s. of Eq. (1) and (2) represent the collision terms. $C_{pe}^{(C)\mu} = -C_{ep}^{(C)\mu}$ is the collision term for the Coulomb scattering between protons and electrons. This term leads to the diffusion of magnetic field and can be neglected in the highly conducting medium in early universe. On the other hand, the collision terms for the Thomson scattering between protons (electrons) and photons are expressed as $C_{p(e)\gamma}^{(T)\mu}$. Because the collision term for the protons can be neglected compared to that for the electrons, difference in velocities of protons and electrons will be induced which leads to electric current. This electric current becomes a source for magnetic field.

Now we evaluate the collision term for the Thomson scattering: $\gamma(p_i) + e^-(q_i) \rightarrow \gamma(p'_i) + e^-(q'_i)$, where the quantities in the parentheses denote the particle momenta. The collision integral $C_{\gamma e}^{(T)}[f(p_i)]$ for this scattering is given as [19],

$$C_{\gamma e}^{(T)}[f(p_i)] = \int \frac{d^3 p}{(2\pi)^3} p^i C_{\gamma e}^{(T)}[f(p_i)] = \frac{4\sigma_T \rho_\gamma n_e}{3} \left[(u_e^i - u_\gamma^i) + \frac{1}{8} u_{ej} \Pi_\gamma^{ij} \right], \quad (5)$$

where σ_T is the cross section of the Thomson scattering. Here moments of the distribution functions are given by

$$\int \frac{d^3 p}{(2\pi)^3} p f_\gamma(p_i) = \rho_\gamma, \quad (6)$$

$$\int \frac{d^3 p}{(2\pi)^3} p^i f_\gamma(p_i) = \frac{4}{3} \rho_\gamma u_\gamma^i, \quad (7)$$

$$\int \frac{d^3 p}{(2\pi)^3} p^i f_e(p_i) = \rho_e u_e^i, \quad (8)$$

$$\int \frac{d^3 p}{(2\pi)^3} p^{-1} p^i p^j f_\gamma(p_i) = \frac{1}{6} \rho_\gamma \Pi_\gamma^{ij} + \frac{1}{3} \rho_\gamma \delta^{ij}, \quad (9)$$

where Π_γ^{ij} is photon anisotropic stress. It should be noted that the collision term (5) was obtained non-perturbatively with respect to the cosmological perturbation.

Altogether, the Euler equations for protons and electrons are written as

$$m_p n u_p^\mu u_{p;\mu}^i - e n u_p^\mu F_\mu^i = 0, \quad (10)$$

$$m_e n u_e^\mu u_{e;\mu}^i + e n u_e^\mu F_\mu^i = -\frac{4\sigma_T \rho_\gamma n}{3} \left[(u_e^i - u_\gamma^i) + \frac{1}{8} u_{ej} \Pi_\gamma^{ij} \right], \quad (11)$$

where m_p is the proton mass, and the pressure of proton and electron fluids are neglected. We also assumed the charge neutrality: $n = n_e \sim n_p$. Subtracting Eq. (10) multiplied by m_e from Eq. (11) multiplied by m_p , we obtain

$$\begin{aligned} & -\frac{m_p m_e}{e} \left[n u^\mu \left(\frac{j^\mu}{n} \right)_{;\mu} + j^\mu \left(\frac{m_p - m_e}{m_p + m_e} \frac{j^\mu}{en} - u^i \right)_{;\mu} \right] \\ & + en(m_p + m_e) u^\mu F_{\mu}^i - (m_p - m_e) j^\mu F_{\mu}^i \\ & = -\frac{4m_p \sigma_T \rho_\gamma n}{3} \left[(u_e^i - u_\gamma^i) + \frac{1}{8} u_{ej} \Pi_\gamma^{ij} \right], \end{aligned} \quad (12)$$

where u^μ and j^μ are the center-of-mass 4-velocity of the proton and electron fluids and the net electric current, respectively, defined as

$$u^\mu \equiv \frac{m_p u_p^\mu + m_e u_e^\mu}{m_p + m_e}, \quad j^\mu \equiv en(u_p^\mu - u_e^\mu). \quad (13)$$

Noting the Maxwell equations $F_{;\nu}^{\mu\nu} = j^\mu$, we see that the first term in the l.h.s. of Eq. (12) are suppressed, compared to the second term, by a factor [21]

$$\frac{c^2}{L^2 \omega_p^2} \sim 3 \times 10^{-34} \left(\frac{10^3 \text{cm}^{-3}}{n} \right) \left(\frac{1 \text{kpc}}{L} \right)^2, \quad (14)$$

where c is the speed of light, L is a characteristic length of the system and $\omega_p = \sqrt{4\pi n e^2 / m_e}$ is the plasma frequency. Second order vector perturbations are contained in the covariant derivative of the electric current and were evaluated in [16]. They obtained the current magnetic field of order 10^{-29} G at 1Mpc scale. As we will see below, this is much smaller than a value we obtain in this paper, which justifies the neglect of the first term in the l.h.s. of Eq. (12).

The third term in the l.h.s. of Eq. (12) is the Hall term which can also be neglected because the Coulomb coupling between protons and electrons is so tight that $|u^i| \gg |u_p^i - u_e^i|$. Then we have a generalized Ohm's law:

$$u^\mu F_{\mu}^i = -\frac{4\sigma_T \rho_\gamma}{3e} \left[(u_e^i - u_\gamma^i) + \frac{1}{8} u_{ej} \Pi_\gamma^{ij} \right] \equiv C^i. \quad (15)$$

Now we derive the evolution equation for magnetic field. This can be obtained from the Bianchi identities $F_{[\mu\nu,\lambda]} = 0$, as

$$\begin{aligned} 0 &= \epsilon^{ijk} u^\mu F_{[jk,\mu]} \\ &= u^\mu B_\mu^i - \frac{1}{u^0} \epsilon^{ijk} C_j u_{,k}^0 + \epsilon^{ijk} C_{j,k} \\ &\quad - (u^i_{,j} B^j - u^j_{,j} B^i) + \frac{u^0_{,j}}{u^0} (B^j u^i - B^i u^j), \end{aligned} \quad (16)$$

where ϵ^{ijk} is the Levi-Civita tensor and $B^i \equiv \epsilon^{ijk} F_{jk}/2$ is comoving magnetic field. We can expand the photon energy density, fluid velocities and photon anisotropic stress as

$$\begin{aligned} \rho_\gamma &= \rho_\gamma^{(0)} + \rho_\gamma^{(1)} + \dots, \quad u_0 = 1 + u_0^{(2)} + \dots, \\ u_i &= u_i^{(1)} + u_i^{(2)} + \dots, \quad \Pi_\gamma^{ij} = \Pi_\gamma^{(1)ij} + \dots, \end{aligned} \quad (17)$$

where the superscripts (i) denote the order of expansion. Remembering that B^i is a small quantity, we see that most of

the terms in Eq. (16), other than the first and third terms, can be neglected. Thus we obtain

$$\begin{aligned} \dot{B}^i &\sim -\epsilon^{ijk} C_{j,k} \\ &\sim \frac{4\sigma_T \rho_\gamma^{(0)}}{3e} \epsilon^{ijk} \left[\frac{\rho_{\gamma,k}^{(1)}}{\rho_\gamma^{(0)}} \left(u_{ej}^{(1)} - u_{\gamma j}^{(1)} \right) + \left(u_{ej,k}^{(2)} - u_{\gamma j,k}^{(2)} \right) \right. \\ &\quad \left. + \frac{1}{8} \left(u_{el,k}^{(1)} \Pi_{\gamma j}^{(1)} + u_{el}^{(1)} \Pi_{\gamma j,k}^{(1)} \right) \right], \end{aligned} \quad (18)$$

where the dot denotes a derivative with respect to the cosmic time, and we used the fact that there is no vorticity in the linear order: $\epsilon^{ijk} u_{j,k}^{(1)} = 0$. The contributions of the first two terms in Eq. (18) were first noticed in [17]. From this expression, we see that magnetic field cannot be generated in the linear order. Here it should be noted that the velocity of electron fluid can be approximated to the center-of-mass velocity at this order, $u_e^{(1)i} \sim u^{(1)i}$.

3. Evaluation of generated magnetic field

As we saw in Eq. (18), there are three main contributions to the generation of magnetic fields (1) baryon-photon slip term, (2) vorticity difference term, and (3) anisotropic pressure term. These terms derive from the fact that electrons are pushed by photons through Compton scattering when there exist velocity differences between them, or anisotropic pressure from photons. Here we now derive the power spectrum of magnetic fields, and then perform a numerical calculation to evaluate it. The power spectrum of magnetic fields $S(k)$ is defined as $\langle |B(k)|^2 \rangle \equiv S(k)$, where \mathbf{k} is the wave vector, $B(k)$ is Fourier component of magnetic fields. $S(k)$ represents the expected variance of magnetic fields $B(k)$, from which the component of the field with characteristic scale λ can be derived through $B_\lambda \approx \sqrt{k^3 S(k) / (2\pi^2)}$ with $\lambda = 2\pi/k$. The power spectrum of magnetic fields $S(k)$ is defined by the expected variance of the Fourier component of magnetic fields $B(k)$ as $S(k) \equiv \langle |B(k)|^2 \rangle$, where \mathbf{k} is the wave vector. The component of the field with characteristic scale λ can then be derived through $B_\lambda \approx \sqrt{k^3 S(k) / (2\pi^2)}$ with $\lambda = 2\pi/k$. We consider a standard cosmological model which consists of photons, baryons, cold dark matter, neutrinos, and cosmological constant, fixing all the cosmological parameters to the standard values. The density perturbations of them were solved numerically in a range of scales from 10 kpc up to 10 Gpc, and they were then integrated to obtain $S(k)$. We found that the field strength of generated magnetic fields at cosmological recombination can be as large as $10^{-16.8}$ G at 1Mpc comoving scale, and it becomes even larger at smaller scales ($10^{-12.8}$ G at 10 kpc) (Fig. 2). After cosmological recombination, no magnetic fields would be generated, since most of electrons were combined into hydrogen atoms and Compton scattering was no longer efficient. This means that the fields have an amplitude of $10^{-22.8}$ G at 1Mpc ($10^{-18.8}$ G at 10kpc) at present because magnetic fields decay adiabatically as the universe expands after their generation. It is large enough to seed the galactic magnetic

fields required by the dynamo mechanism, which is typically of the order of $10^{-20} \sim 10^{-30}$ G at around the 10 kpc scale 1.

Generated magnetic fields monotonically increase towards smaller scales on a range of scales in our calculation. We found that the field has the spectrum $S(k) \propto k^4$ at scales larger than $\sim 10^{2.5}$ Mpc, which corresponds to super-horizon scales at recombination, $S(k) \propto k^0$ at intermediate scales ($10^{2.5}$ Mpc $< \lambda < 10^{1.5}$ Mpc), and $S(k) \propto k^1$ at scales smaller than ~ 10 Mpc, where the contribution from the anisotropic stress of photons dominates. This means that the field strength B is proportional to k^2 at the scales smaller than 1 Mpc. If the primordial power spectrum of density fluctuations is given by a simple power law as predicted by inflation, our result implies that magnetic fields with the strength $B \approx 10^{-12.8}$ G unavoidably arise on 100 pc comoving scale at $z \approx 10$. This value is interesting for the evolution of structures in the high-redshift universe, since those magnetic fields would be strong enough for magneto-rotational instability in the accretion disks surrounding very first stars (Population III stars) to be triggered, and affect the transport of their angular momentum [27]. The transport of angular momentum plays an important role for mass accretion process onto protostars. Since typical mass scale of the Population III stars is a key [27] for the early reionization and chemical evolution of the universe, cosmologically generated magnetic fields should be one of essential ingredients in the model of the structure formation in the high-redshift universe.

Since magnetic field creation mainly occurs when the modes of density perturbations with the corresponding scale enter the cosmic horizon and become causally connected [20], the magnetic fields should exist at small scales below ~ 10 Mpc even where the Silk damping effect by photons' diffusion has swept away the density perturbations at the last scattering epoch. Thus, in principle, the detection of magnetic fields below ~ 10 Mpc scales calculated here would tell us about density perturbations in photons (and baryons) in the early universe even at scales smaller than the diffusion scale at recombination. In this sense, the magnetic field generated in this mechanism can be regarded as a fossil of density perturbations in the early universe, whose signature in photons and baryons has been lost. This result provides consequently a possibility of probing observations on how density perturbations in photons had evolved and been swept away at these small scales where no one can, in principle, probe directly through photons.

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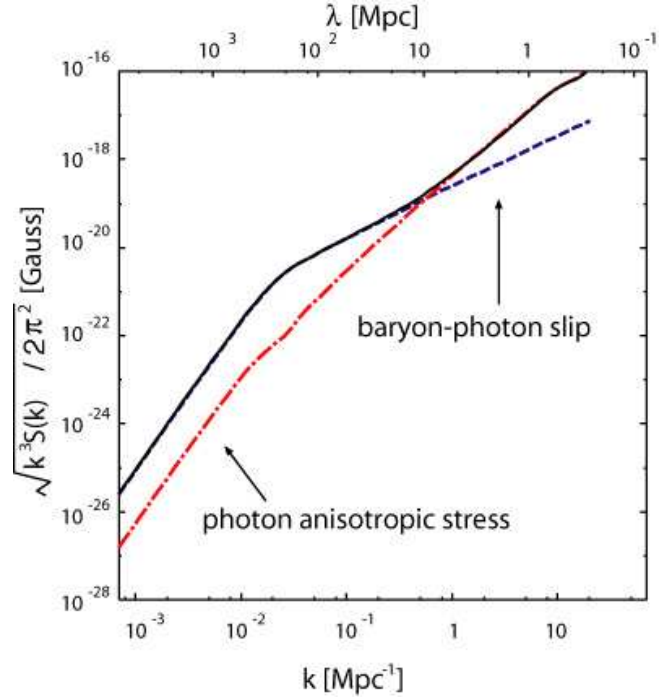


Fig. 2. Spectrum of magnetic fields $S(k)$ generated from cosmological perturbations at cosmological recombination. (We plot $\sqrt{k^3 S(k)}$ instead of $S(k)$ in order to measure in units of Gauss.) Blue dashed and red dot-dashed lines show contributions from the baryon-photon slip and photon's anisotropic stress, respectively. The spectrum decays as k^4 at scales larger than that of the cosmic horizon at cosmological recombination. At small scales, the contribution from the anisotropic stress of photons dominates and the spectrum has a slope proportional to k .

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